TRINITY COLLEGE

Year 12 Methods Units 3,4
Test 2 2021

Section 1 Calculator Free

Area, Fundamental Theorem, Exponential Function

STUDENT'S NAME ______________________

DATE: Thursday 25 March **TIME:** 25 minutes **MARKS:** 25

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (4 marks)

Determine $\frac{d}{dx}$ for each of the following.

(a) $e^{1-x}(x^e - 8)$ $\frac{d}{dx} = (x^e - 8)(-e^{1-x}) + e^{1-x}\, e x^{e-1}$ [2]

(b) $\frac{d}{dx}\int_{-1}^{x} \frac{e^{\pi} - e^{t+1}}{\sqrt{1+t}}\, dt = \frac{e^{\pi} - e^{x+1}}{\sqrt{1+x}}$ [2]

2. (11 marks)

(a) Determine each of the following.

(i) $\int 3xe^{x^2-6}dx$ $= \frac{3}{2}\int 2x e^{x^2-6} dx$ [2]

$= \frac{3}{2} e^{x^2-6} + c$

(ii) $\int \frac{8e^{2x}+e^{-x+1}}{e^{-x}}dx$ $= 8\int e^{3x} dx + \int e \, dx$ [3]

$= \frac{8}{3}\int 3e^{3x} dx + ex + c$

$= \frac{8}{3} e^{3x} + ex + c$

(b) Given $\frac{dP}{dt} = e^{4-2t}$ determine an expression for P if $P = \frac{e^2}{2}$ when $t = 1$ [3]

$P = \int e^{4-2t} dt$

$P = -\frac{1}{2}\int -2e^{4-2t} dt$

$P = -\frac{1}{2} e^{4-2t} + c$

$\frac{e^2}{2} = -\frac{1}{2} e^{4-2} + c$

$e^2 = c$

$P = \frac{-e^{4-2t}}{2} + e^2$

(c) Evaluate $\int_0^1 \frac{d}{dx}(\frac{x^3}{x^2+1})dx$ [3]

$= \left[\frac{x^3}{x^2+1} \right]_0^1$

$= \frac{1}{2} - \frac{0}{1}$

$= \frac{1}{2}$

3. (6 marks)

(a) Determine $\frac{d}{dx}xe^{2x}$ $= e^{2x} + 2xe^{2x}$ [2]

Hence or otherwise evaluate exactly

(b) $\int_0^1 2xe^{2x}dx$ [4]

$$\frac{d}{dx} xe^{2x} = e^{2x} + 2xe^{2x}$$

$$\int_0^1 \frac{d}{dx} xe^{2x}\,dx = \int_0^1 e^{2x}\,dx + \int_0^1 2xe^{2x}\,dx$$

$$\left[xe^{2x}\right]_0^1 = \frac{1}{2}\int_0^1 2e^{2x} + \int_0^1 2xe^{2x}\,dx$$

$$e^2 - 0 = \left[\frac{1}{2}e^{2x}\right]_0^1 + \int_0^1 2xe^{2x}\,dx$$

$$e^2 = \frac{e^2}{2} - \frac{1}{2} + \int 2xe^{2x}\,dx$$

$$\frac{e^2}{2} + \frac{1}{2} = \int 2xe^{2x}\,dx$$

4. (4 marks)

Determine and classify all stationary points of the curve $y = x^2e^x$.

$y' = 2xe^x + x^2e^x$

$2xe^x + x^2e^x = 0$

$e^x(2x + x^2) = 0$

$e^x = 0$ $\quad\quad$ $2x + x^2 = 0$

N.S $\quad\quad$ $x(2+x) = 0$

$x = 0, -2$

$y'' = 2e^x + 2xe^x + 2xe^x + x^2e^x$

$= 2e^x + 4xe^x + x^2e^x$

$y''\big|_{x=0} = 2$ $\quad\therefore$ MIN $\quad(0,0)$

$y''\big|_{x=-2} = 2e^{-2} - 8e^{-2} + 4e^{-2}$

< 0 $\quad\therefore$ MAX $\quad\left(-2, \frac{4}{e^2}\right)$

TRINITY COLLEGE

Year 12 Methods Units 3,4
Test 2 2021

Section 2 Calculator Assumed

Area, Fundamental Theorem, Exponential Function

STUDENT'S NAME ______________________

DATE: Thursday 25 March **TIME:** 25 minutes **MARKS**: 28

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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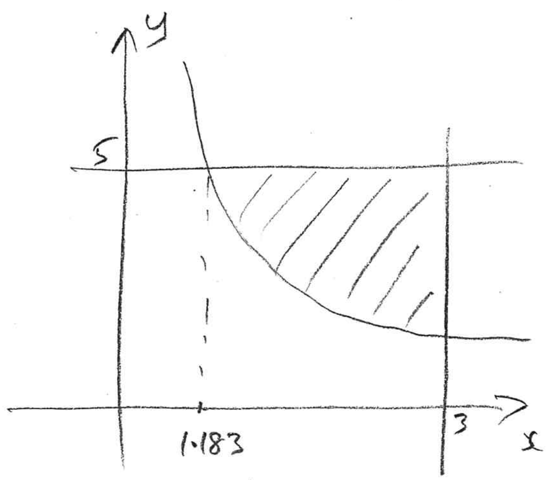
5. (4 marks)

Determine the area enclosed by $x = 3$, $y = 5$ and $y = \dfrac{2}{x^2 - 1}$.

$$\text{AREA} = (3 - 1.183)5 - \int_{1.183}^{3} \frac{2}{x^2-1}\,dx$$

$$= 9.1 - 1.8$$

$$= 7.3$$



6. (6 marks)

Scientists study a population of mice over a ten week period and conclude that the population is increasing at a rate given by $R'(t) = 28e^{0.15t}$ where t is the number of weeks since the study began and an initial population of 30 mice.

(a) What is the change in the population in the seventh week of the study? [2]

$$\int_6^7 28e^{0.15t}\,dt \approx 74$$

(b) What is the average weekly increase in mice over the ten week period? [2]

$$\frac{\int_0^{10} 28e^{0.15t}\,dt}{10} \approx 65/\text{WEEK}$$

(c) How long does it take for the mice population to reach 150? [2]

$$\int_0^k 28e^{0.15t}\,dt = 120$$

$$\frac{28}{0.15}\int_0^k 0.15e^{0.15t}\,dt = 120$$

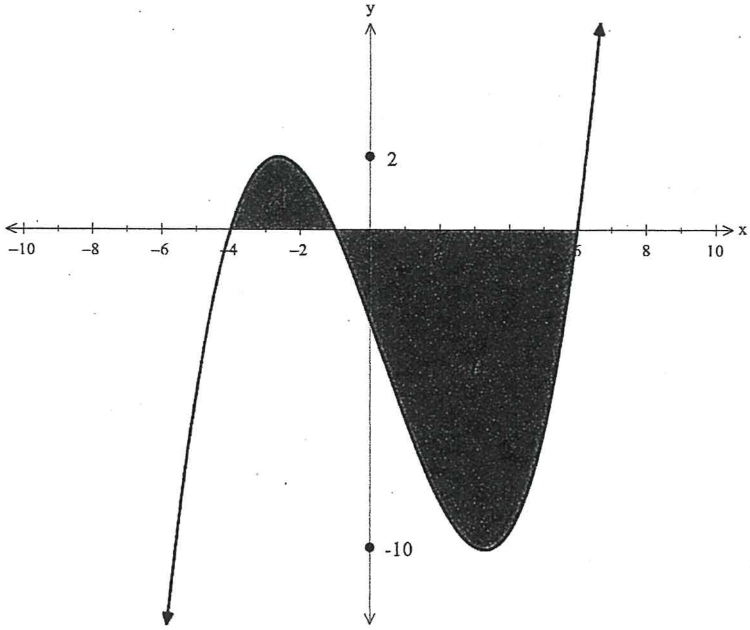
$$\left[e^{0.15t}\right]_0^k = \frac{120 \times 0.15}{28}$$

$$e^{0.15k} - 1 = 0.643$$

$$e^{0.15k} = 1.643$$

$$k = 3.3 \text{ WEEKS}$$

7. (10 marks)



In the diagram above showing the graph of $y = f(x)$, the shaded region A has an area of 5 square units. Shaded region B has an area of 30 square units.

Using the information above, determine

(a) $\int_{-4}^{6} 2f(x)dx$ $\quad -50$ [2]

(b) $\int_{-4}^{-1} (f(x)+2)dx$ [3]

$= \int_{-4}^{1} f(x)\,dx + \int_{-4}^{-1} 2\,dx$

$= 5 + [2x]_{-4}^{-1}$

$= 5 + (-2 - (-8))$

$= 11$

(c) $\int_{-3}^{2} f(2x+2)dx$ [3]

$= \int_{2}^{-3} f(2(x+1))\,dx = \frac{5}{2} - \frac{30}{2}$

$= -\frac{25}{2}$

(d) $\int_{-1}^{6} f'(x)dx$ [2]

$= f(6) - 6(-1)$

$= 0 - 0$

$= 0$

8. (4 marks)

A curve for which $\frac{dy}{dx} = -e^{kx}$, where k is a constant, is such that the tangent at $(1, -e^3)$ passes through the origin.

(a) Determine the gradient of the tangent. [1]

$$m = \frac{-e^3}{1}$$

(b) Determine the equation of the curve. [3]

$(x=1)$ $$-e^{kx} = -e^{3}$$

$$\therefore k = 3$$

$$y = \int -e^{3x}\,dx$$

$$= -\frac{1}{3}\int 3e^{3x}\,dx$$

$$y = -\frac{e^{3x}}{3} + c$$

$(1, -e^3)$ $$-e^3 = -\frac{e^{3x}}{3} + c$$

$$-e^3 = -\frac{e^3}{3} + c$$

$$-\frac{2e^3}{3} = c$$

$$y = -\frac{e^{3x}}{3} - \frac{2e^3}{3}$$

9. (4 marks)

A continuous function $f(x)$ is increasing on the interval $0 < x < 2$ and decreasing on the interval $2 < x < 6$. Some of its values are given in the table below.

x	0	1	2	3	4	5	6
$f(x)$	5	16	27	23	16	0	-12

The function $F(x)$ is defined for $0 \le x \le 6$ by $F(x) = \int_0^x f(t)dt$.

(a) At which value of x in the interval $0 \le x \le 6$ is $F(x)$ the greatest? Justify your answer. [2]

$x = 5$ AREA KEEPS INCREASING UP TO WHERE $f(x) = 0$

(b) At which value of x in the interval $0 \le x \le 6$ is $F'(x)$ the greatest? Justify your answer. [2]

$x = 2$ AS INCREASING FUNCTION UP TO $x = 2$ AND THEN DECREASES.